

Graph theory
Final exam
2013.12.11.

1. Define the following notions:
 - a) perfect graph. (12 points)
 - b) vertex cover. (12 points)
 - c) Ramsey number $R(k, l)$. (12 points)
 - d) Turán graph $T(n, r)$. (12 points)
 - e) k -connected graph. (12 points)

2. Formulate the following theorems:
 - a) Strong perfect graph theorem. (20 points)
 - b) Turán's theorem. (20 points)

3. State and prove Mantel's theorem. (40 points)

4. There are 49 pairwise non-isomorphic graphs that can be obtained from a cycle of length 101 by adding exactly one diagonal to the cycle. How many of these graphs are perfect? (40 points)

5. Determine the maximum number of edges of a 7 vertex simple graph that does not contain an even cycle. (40 points)

6. Let G be a simple graph on 12 vertices, such that among any three vertices there are (at least) two neighbours. Prove that G contains a matching of size 5. (40 points)

7. Find a simple graph G on 9 vertices, such that $\omega(G) = 3$ and $\alpha(G) = 3$ (and prove that for your graph both equations hold.) (40 points)

8. Let the vertices of a graph G be the numbers $1, 2, \dots, 100$. There is an edge between different numbers i and j if $i \mid j$ or $j \mid i$. Determine $\tau(G)$. (40 extra points)