

Big-Oh notation

1. Decide in all possible cases whether $f_i(n) = O(f_j(n))$ is true or not if

$$f_1(n) = 11n^2, \quad f_2(n) = 8n^2 \log n, \quad f_3(n) = n^2 + 100000.$$

2. (a) Let's suppose that $f(n) = O(n^2)$ and $g(n) = \Theta(n^3)$. Is it true that $f(n) = O(g(n))$?
(b) Let's suppose that $f(n) = O(n^3)$ and $g(n) = \Theta(n^2)$. Is it true that $g(n) = O(f(n))$?
(c) Let's suppose that $f(n) = O(n^3)$ and $g(n) = O(n^2)$. Is it true that $g(n) = O(f(n))$? Is it possible that $f(n) = O(g(n))$?
3. Let's suppose that $f(n)$ and $g(n)$ are functions with non-negative values. Prove that

$$\max(f(n), g(n)) = \Theta(f(n) + g(n))$$

4. Give a linear algorithm (ie. whose running time is $O(n)$) using only comparisons to find the maximum among n different numbers. What is the precise number of comparisons we have to perform to find the maximum?